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Relationships between common irrigation application uniformity indicators

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Abstract The coefficient of uniformity, CU, and the distribution uniformity, DU, are perhaps the two most common indicators of irrigation application uniformity, especially for pressurized irrigation methods. The magnitude of CU is usually greater than that of DU, but this is not the case for all data sets, as has been observed in practice by irrigation engineers and researchers. This paper describes the conditions under which CU > DU, and vice versa, proving that either situation can occur in practice. A comparison of some alternative measures of irrigation application uniformity is also compared using two data sets from agricultural sprinklers operating at different pressures.

Introduction

Performance indicators of irrigation water application uniformity are useful to compare irrigation methods, modifications to irrigation methods, and historical trends in the management of any given method. With such comparisons, areas for improvement can be identified and the water can be better managed. Many publications on the topic of irrigation application uniformity can be found. For example, Barragan et al. (2006) describe procedures for determining emission uniformity in micro-irrigation emitters and Burt (2004) discusses field evaluation methods for drip and micro-spray application uniformity. Burt et al. (1997) consider different irrigation performance measures.

Keller and Bliesner (2000) provide a definition of the Christiansen (1942) coefficient of uniformity as follows:

$$\mathrm{CU}_{\mathrm{true}} = 1 - \frac{\sum |d - m|}{\sum d} \qquad (1)$$

where $\mathrm{CU}_{\mathrm{true}}$ is expressed in Eq. 1 as a fraction (CU $\leq$ 1); d are the individual measured depths or volumes of water; and m is the average of all d values. The summations in Eq. 1 are for all values of d in the data set. It is noted that for very poor water application uniformities, the value of CU can be less than zero.

Another common index for application uniformity is DU, or distribution uniformity (Merriam and Keller 1978). This index is calculated as the ratio of the average depth (or discharge) of the low ¼ of the values to the average of all values. Thus, the values of d, as defined for Eq. 1, are ranked from low to high, and if each value represents the same unit field area, the first 25% of the values are averaged and divided by the average of all the values. The ratio is typically multiplied by 100 to obtain a percent, as is often done for CU as well.

For most uniformity data sets, the magnitude of DU is less than that of CU, leading some irrigation engineers and specialists to assume that this is always the case, and that there is an error when it is not. However, even though it might not be obvious, there are valid cases when DU > CU and the conditions for this occurrence are defined in this paper.

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Comparison of CU and DU values

Irrigation water application uniformity is usually based on measured water depths or measured discharges. For example, suppose that measured catch-can depths from a sprinkler evaluation are ranked from high to low. Let A, B, C, and D be the average of the measured depth values in these four data groups. That is,

A = the average of the d values in the highest quarter
B = the average of the d values in the second highest quarter
C = the average of the d values in the second lowest quarter
D = the average of the d values in the lowest quarter

Then, it follows that

$$A \geq B \geq C \geq D \tag{2}$$

As described above, the distribution uniformity, DU, is defined as the average of the values in the lowest quarter, D, divided by the average of all values. Based on the definitions in Eq. 2,

$$\mathrm{DU} = \frac{D}{\frac{A+B+C+D}{4}} = \frac{4D}{A+B+C+D} \tag{3}$$

where DU is expressed as a fraction ($0 \leq \mathrm{DU} \leq 1$).

Similarly, Christiansen's coefficient of uniformity, $\mathrm{CU}_{\mathrm{true}}$, can be approximated as:

$$\mathrm{CU}_{\mathrm{approx}} = 1 - \frac{\left|A - \frac{A+B+C+D}{4}\right| + \left|B - \frac{A+B+C+D}{4}\right| + \left|C - \frac{A+B+C+D}{4}\right| + \left|D - \frac{A+B+C+D}{4}\right|}{A+B+C+D} \tag{4}$$

or,

$$\mathrm{CU}_{\mathrm{approx}} = 1 - \frac{|3A-(B+C+D)| + |3B-(A+C+D)| + |3C-(A+B+D)| + |3D-(A+B+C)|}{4(A+B+C+D)} \tag{5}$$

Considering the terms in Eq. 5, from Eq. 2, it follows that

$$3A - (B+C+D) \geq 0 \tag{6}$$

Thus,

$$|3A-(B+C+D)| = 3A-(B+C+D) \tag{7}$$

Also,

$$3D - (A+B+C) \leq 0 \tag{8}$$

which means that

$$|3D-(A+B+C)| = A+B+C-3D \tag{9}$$

Introduce Eqs. 7 and 9 into Eq. 5,

$$\mathrm{CU}_{\mathrm{approx}} = 1 - \frac{3A-(B+C+D) + |3B-(A+C+D)| + |3C-(A+B+D)| + (A+B+C) - 3D}{4(A+B+C+D)} \tag{10}$$

or,

$$\mathrm{CU}_{\mathrm{approx}} = 1 - \frac{4A - 4D + |3B-(A+C+D)| + |3C-(A+B+D)|}{4(A+B+C+D)} \tag{11}$$

The value $\mathrm{CU}_{\mathrm{approx}}$ is calculated using the average values of four data groups, $A - D$. It is not necessarily the same as the $\mathrm{CU}_{\mathrm{true}}$ value calculated by Eq. 1. However, the numerator of the ratio in Eq. 4 is greater than or equal to the numerator in the ratio shown in Eq. 1. Consequently, except in special cases, $\mathrm{CU}_{\mathrm{approx}}$ will be greater than $\mathrm{CU}_{\mathrm{true}}$. As shown below, sometimes $\mathrm{DU} \geq \mathrm{CU}_{\mathrm{approx}}$, and in these cases, DU will also be greater in magnitude than $\mathrm{CU}_{\mathrm{true}}$ for the same conditions.

Considering the absolute value terms, there are four conceivable situations, or cases, when calculating $\mathrm{CU}_{\mathrm{approx}}$ using Eq. 11, and given the relationships defined in Eq. 2:

$$\text{Case 1}: 3B-(A+C+D) \geq 0 \text{ and } 3C - (A+B+D) \geq 0 \tag{12}$$

$$\text{Case 2}: 3B-(A+C+D) \geq 0 \text{ and } 3C - (A+B+D) \leq 0 \tag{13}$$

$$\text{Case 3}: 3B-(A+C+D) \leq 0 \text{ and } 3C - (A+B+D) \leq 0 \tag{14}$$

$$\text{Case 4}: 3B - (A + C + D) \leq 0 \text{ and } 3C - (A + B + D) \geq 0 \tag{15}$$

Case 1 occurs when $B + C \geq A + D$, and case 2 occurs when $B \geq C$. Case 3 is true when $A + D \geq B + C$, and case 4 will never occur (as proven below). For case 1, referring to Eq. 11:

$$\begin{aligned}\mathrm{CU_{approx}} &= 1 - \frac{4A - 4D + 3B - (A + C + D) + 3C - (A + B + D)}{4(A + B + C + D)} \\ &= \frac{A + B + C + 5D}{2(A + B + C + D)}\end{aligned} \tag{16}$$

In this scenario, subtracting Eq. 3 from Eq. 16,

$$\mathrm{CU_{approx}} - \mathrm{DU} = \frac{A + B + C + 5D}{2(A + B + C + D)} - \frac{4D}{A + B + C + D} \tag{17}$$

or,

$$\mathrm{CU_{approx}} - \mathrm{DU} = \frac{A + B + C - 3D}{2(A + B + C + D)} \tag{18}$$

Then, given that $A \geq B \geq C \geq D \geq 0$, both the numerator and denominator must be greater than or equal to zero (but the denominator is never equal to zero in any practical case), resulting in $\mathrm{CU_{approx}} \geq \mathrm{DU}$.

For case 2, referring to Eq. 11:

$$\begin{aligned}\mathrm{CU_{approx}} &= 1 - \frac{4A - 4D + 3B - (A + C + D) - 3C + (A + B + D)}{4(A + B + C + D)} \\ &= \frac{2(C + D)}{A + B + C + D}\end{aligned} \tag{19}$$

In this second scenario, subtracting Eq. 3 from Eq. 19,

$$\begin{aligned}\mathrm{CU_{approx}} - \mathrm{DU} &= \frac{2(C + D)}{A + B + C + D} - \frac{4D}{A + B + C + D} \\ &= \frac{2(C - D)}{A + B + C + D}\end{aligned} \tag{20}$$

Then, given that in Eq. 20, both the numerator and the denominator must be greater than or equal to zero, it is again found that $\mathrm{CU_{approx}} \geq \mathrm{DU}$.

Similarly, for case 3,

$$\begin{aligned}\mathrm{CU_{approx}} &= 1 - \frac{4A - 4D - 3B + (A + C + D) - 3C + (A + B + D)}{4(A + B + C + D)} \\ &= \frac{3(B + C + D) - A}{2(A + B + C + D)}\end{aligned} \tag{21}$$

Then, subtracting the DU from Eq. 21,

$$\begin{aligned}\mathrm{CU_{approx}} - \mathrm{DU} &= \frac{3(B + C + D) - A}{2(A + B + C + D)} - \frac{4D}{A + B + C + D} \\ &= \frac{3(B + C) - (A + 5D)}{2(A + B + C + D)}\end{aligned} \tag{22}$$

In Eq. 22, the denominator is always greater than or equal to zero (in practice, it will never equal zero). Thus, if $3(B + C) \geq A + 5D$, then $\mathrm{CU_{approx}} \geq \mathrm{DU}$, as in cases 1 and 2. However, if $3(B + C) \leq A + 5D$, then $\mathrm{DU} \geq \mathrm{CU_{approx}}$. This is the one case in which the value of DU can exceed the value of $\mathrm{CU_{approx}}$.

The fourth case is not possible, and the reason is given as follows. If $3B - (A + C + D) \leq 0$ and $3C - (A + B + D) \geq 0$, then these two inequalities must be true:

$$A + C + D \geq 3B \tag{23}$$

and

$$3C \geq A + B + D \tag{24}$$

Adding Eqs. 23 and 24,

$$A + C + D + 3C \geq 3B + A + B + D \tag{25}$$

which can be simplified to:

$$C \geq B \tag{26}$$

But Eq. 26 violates Eq. 2, so it is not a valid occurrence and case 4 can be discarded.

Sample calculations

Two sprinkler data sets with ranked catch-can depth values are shown in Table 1. These two data sets provide examples of cases 1 and 3, as described above. For data set no. 1, $A = 6.52$, $B = 6.01$, $C = 5.29$, and $D = 2.63$, as shown in Table 1. Thus,

$$[B + C = 11.30] > [A + D = 9.15] \tag{27}$$

Consequently, this data set is described by case 1, whereby $\mathrm{CU_{approx}} > \mathrm{DU}$. $\mathrm{CU_{approx}}$ can be calculated using Eq. 16:

$$\mathrm{CU_{approx}} = \frac{6.52 + 6.01 + 5.29 + 5(2.63)}{2(6.52 + 6.01 + 5.29 + 2.63)} = 0.757 \tag{28}$$

$\mathrm{CU_{true}}$ and DU can be calculated using Eqs. 1 and 3, respectively, giving $\mathrm{CU_{true}} = 75.3\%$ and $\mathrm{DU} = 51.5\%$. Figure 1 provides a graphical comparison of several different measures of irrigation application uniformity, as described above, for data set no. 1.

For data set no. 2, $A = 2.58$, $B = 1.50$, $C = 1.28$, and $D = 1.16$, as shown in Table 1. Thus,

$$[3(B + C) = 8.34] < [A + 5D = 8.38] \tag{29}$$

So, this data set is described by one of the two possibilities in case 3, whereby $\mathrm{CU_{approx}} < \mathrm{DU}$. $\mathrm{CU_{approx}}$ can be calculated using Eq. 21:

$$\mathrm{CU_{approx}} = \frac{3(1.50 + 1.28 + 1.16) - 2.58}{2(2.58 + 1.50 + 1.28 + 1.16)} = 0.708 \tag{30}$$

Table 1 Ranked catch values (mm) from two data sets

Data set no. 1				Data set no. 2			
First quarter	Second quarter	Third quarter	Fourth quarter	First quarter	Second quarter	Third quarter	Fourth quarter
1.18	4.97	5.88	6.21	1.11	1.20	1.44	2.14
1.18	4.97	5.88	6.21	1.11	1.20	1.44	2.14
1.18	4.97	5.88	6.21	1.11	1.20	1.44	2.24
1.18	4.97	5.88	6.21	1.11	1.20	1.44	2.24
1.54	4.97	5.89	6.21	1.14	1.21	1.44	2.33
1.54	4.97	5.89	6.21	1.14	1.21	1.44	2.33
1.54	4.97	5.89	6.21	1.14	1.21	1.44	2.33
1.54	4.97	5.89	6.21	1.14	1.21	1.44	2.33
1.54	5.07	5.89	6.29	1.14	1.21	1.44	2.33
1.54	5.07	5.89	6.29	1.14	1.21	1.44	2.33
1.54	5.07	5.89	6.29	1.14	1.21	1.44	2.33
1.54	5.07	5.89	6.29	1.14	1.21	1.44	2.33
2.02	5.07	6.03	6.29	1.16	1.26	1.46	2.48
2.02	5.07	6.03	6.29	1.16	1.26	1.46	2.48
2.02	5.07	6.03	6.29	1.16	1.26	1.46	2.58
2.02	5.07	6.03	6.29	1.16	1.26	1.46	2.58
3.09	5.31	6.03	6.70	1.16	1.26	1.46	2.58
3.09	5.31	6.03	6.70	1.16	1.26	1.46	2.58
3.09	5.31	6.03	6.70	1.16	1.26	1.46	2.58
3.09	5.31	6.03	6.70	1.16	1.26	1.46	2.58
3.09	5.42	6.09	6.70	1.16	1.27	1.47	2.65
3.09	5.42	6.09	6.70	1.16	1.27	1.47	2.65
3.09	5.42	6.09	6.70	1.16	1.27	1.47	2.75
3.09	5.42	6.09	6.70	1.16	1.27	1.47	2.75
3.50	5.42	6.09	6.70	1.17	1.27	1.47	2.75
3.50	5.42	6.09	6.70	1.17	1.27	1.47	2.75
3.50	5.42	6.09	6.70	1.17	1.27	1.47	2.75
3.50	5.42	6.09	6.70	1.19	1.27	1.47	2.75
3.50	5.50	6.11	6.79	1.19	1.42	1.51	2.90
3.50	5.50	6.11	6.79	1.19	1.42	1.52	2.90
3.50	5.50	6.11	6.79	1.19	1.42	1.52	2.90
3.50	5.50	6.11	6.79	1.19	1.42	1.52	2.90
4.23	5.88	6.11	6.79	1.19	1.42	1.55	2.90
4.23	5.88	6.11	6.79	1.19	1.42	1.65	2.90
4.23	5.88	6.11	6.79	1.19	1.42	1.89	2.90
4.23	5.88	6.11	6.79	1.19	1.42	2.00	2.90
$D = 2.63$	$C = 5.29$	$B = 6.01$	$A = 6.52$	$D = 1.16$	$C = 1.28$	$B = 1.50$	$A = 2.58$

CU_{true} and DU can be calculated using Eqs. 1 and 3, respectively, giving $CU_{true} = 0.703$ and $DU = 0.711$. In this example, DU is only slightly greater than CU (also see Fig. 2). Table 2 summarizes the comparison of statistical indicators between data sets 1 and 2.

Figures 1 and 2 show two other application uniformity indicators, as described in Eqs. 31 and 32.

$$m(1 - v) \tag{31}$$

in which the term in parentheses has been referred to as CU_v (Keller 2010) and

$$m\left(1 - \frac{\sum |d - m|}{\sum d}\right) \tag{32}$$

which is the average catch, m, multiplied by CU_{true} (see Eq. 1).

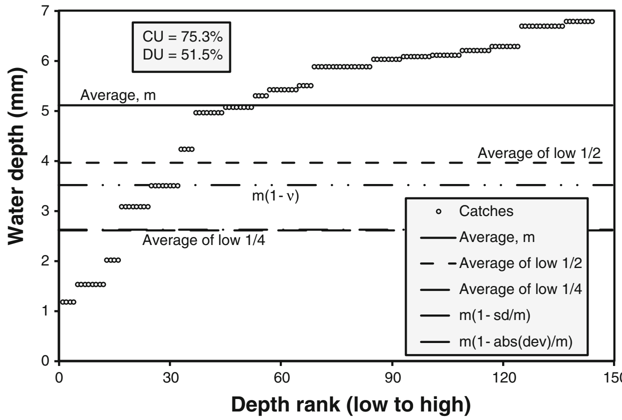

Fig. 1 Graphical comparison of several irrigation application uniformity indicators for data set no. 1

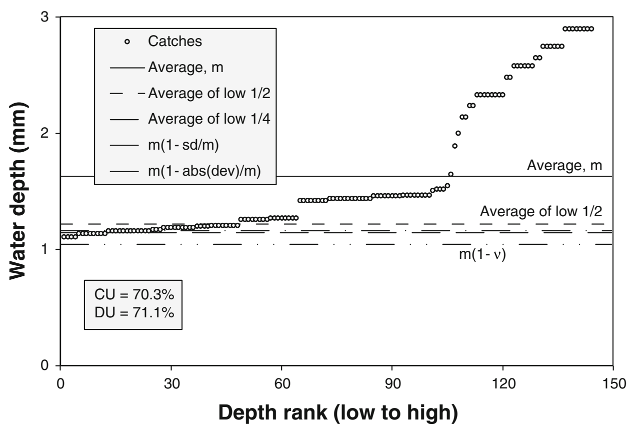

Fig. 2 Graphical comparison of several irrigation application uniformity indicators for data set no. 2

Table 2 Ranked catch values (mm) from two data sets, where "sd" is standard deviation, v is the coefficient of variation, and "Abs(dev)" is the mean absolute difference between each catch value and the average catch

Indicator	Data set	
	No. 1	No. 2
Average catch, m	5.11	1.63
Average of low 1/2	3.96	1.22
Average of low 1/4	2.63	1.16
Standard deviation	1.60	0.58
Avg absolute deviation	1.26	0.48
$m(1-v)$	3.51	1.05
$m(1-\mathrm{Abs(dev)}/m)$	3.85	1.14

Discussion

Data set no. 1 is skewed to the right, and data set no. 2 is skewed to the left. The values of the moment coefficient of skewness, defined as the third moment about the mean divided by the standard deviation cubed (Miller and Freund 1977), are −1.17 and +1.11, respectively, for sets 1 and 2. The highly positive skew of data set no. 2 is one way to explain the fact that CU is less than DU for that set.

Table 3 Three different approximations of CU for data sets 1 and 2

Indicator	Data set	
	No. 1 (%)	No. 2 (%)
CU from Eq. 1	75.3	70.3
CU from Eq. 33	77.5	74.9
CU from Eq. 34	75.0	71.5

There are various alternative ways to approximate the CU value. For example, the average of the low 1/2 of the ranked catch values can be used as follows (with $CU_{\text{low-1/2}}$ as a fraction from 0 to 1):

$$CU_{\text{low}-1/2} = \left(\frac{\text{Avg low }1/2}{m}\right) \tag{33}$$

which is completely analogous to DU, except that the average of the low ½ is used in the numerator instead of the average of the low ¼. Another estimation of CU is based on the equation for a normal distribution:

$$CU \approx 1 - v\sqrt{2/\pi} \tag{34}$$

also as a fraction, where v is the coefficient of variation, defined as the standard deviation divided by the average value, m.

The CU values based on the indicators described above in Eqs. 1, 33, and 34 are all approximately the same (see Table 3) for data set no. 1, even though the distribution of catch values is heavily skewed toward the low end. The distribution for data set no. 2 is strongly skewed toward the high end, but the CU values for this data set are also fairly close to each other. Also, it is noted that $CU_{\text{low-1/2}}$ is significantly greater than DU (which is based on the low 1/4), as expected in all cases.

A number of practical cases in which DU < CU can be identified, including the following:

1. Surface irrigation events in which the water is cutoff too soon, and/or the advance rate slows significantly, resulting in heavy water application at the uphill end of the field, and little or no water application at the downhill side.
2. Sprinkler irrigation in which the sprinklers are spaced too far apart, leaving some dry spots, or where there is a relatively dry area near each sprinkler.
3. Drip irrigation in which many emitters are clogged, or when there are some high-elevation regions in a field that is otherwise mostly level.

Alternatively, the following are some examples of cases in which DU can be found to exceed CU:

1. Surface irrigation events when the soil infiltration rate varies significantly due to variations in soil texture and or structure.
2. Sprinkler irrigation in which the sprinklers are spaced too close to each other, or where there is an area near the sprinklers that receives an overabundance of water, such as when the operating pressure is too high.
3. Drip irrigation where there are some low-elevation regions in a field that is otherwise mostly level.

Summary and conclusions

Two common indicators of irrigation application uniformity are the Christiansen coefficient of uniformity, CU, and the distribution uniformity, DU. When calculated from a single set of measured application depths or discharges, CU is usually larger than DU. But in some cases, the opposite can be true; specifically, when $3(B + C) \leq A + 5D$, $DU \geq CU_{approx}$. And, for any given data set, $CU_{approx} \geq CU_{true}$. Therefore, DU will be greater than CU_{true} when $3(B + C) \leq A + 5D$, which is a valid condition with some data sets. It was also shown how various alternative indicators of irrigation application uniformity can be used to characterize and compare different data sets.

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